Decoherence and multipartite entanglement of non-inertial observers

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Decoherence effect on multipartite entanglement in non-inertial frames is investigated. GHZ state is considered to be shared between the partners with one partner in inertial frame whereas the other two in accelerated frames. One-tangle and π-tangles are used to quantify the entanglement of the multipartite system influenced by phase damping and phase flip channels. It is seen that for phase damping channel, entanglement sudden death (ESD) occurs for $p > 0.5$ in the infinite acceleration limit. On the other hand, in case of phase flip channel, ESD behaviour happens around 50% decoherence. It is also seen that entanglement sudden birth (ESB) does occur in case of phase flip channel. Furthermore, it is seen that effect of environment on multipartite entanglement is much stronger than that of the acceleration of non-inertial frames.

Keywords: Quantum decoherence; multipartite entanglement; non-inertial frames.

I. INTRODUCTION

Quantum entanglement has been recognized as a powerful tool for manipulating information. The emerging field of quantum information processing has opened a new way of using entanglement for performing tasks that are impossible to achieve as efficiently with classical technologies. Quantum information and quantum computation can process multiple tasks which are intractable with classical technologies. Quantum entanglement is no doubt a fundamental resource for a variety of quantum information processing tasks, such as super-dense coding, quantum cryptography and quantum error correction [1-4]. During recent years, two important features entanglement sudden death (ESD) and entanglement Sudden Birth (ESB) has been investigated [5-8]. Yu and Eberly have investigated loss of entanglement in a finite time under the action of pure vacuum noise for

a bipartite system [9,10]. They found that, even though it takes infinite time to complete decoherence locally, the global entanglement may be lost in finite time. Multipartite entangled states can be used to construct variety of information transmission protocols, for example, quantum key distribution [11-12]. For the purposes of quantum communication multipartite entangled states can serve as quantum communication channels in most of the known teleportation protocols [13]. Furthermore, multipartite entanglement displays one of the most fascinating features of quantum theory that is called as nonlocality of the quantum world [14].

However, entangled states are very fragile when they are exposed to environment. Decoherence is the major enemy of entanglement which is responsible for emergence of classical behaviour in quantum systems [15]. Therefore, it could be of great importance to study the deteriorating effect of decoherence in entangled states. Recently, researchers have focused on relativistic quantum information in the field of quantum information science due to conceptual and experimental reasons. In the last few years, much attention had been given to the study of entanglement shared between inertial and non-inertial observers by discussing how the Unruh or Hawking effect will influence the degree of entanglement [16–27]. Implementation of decoherence in non-inertial frames have been investigated for a two-qubit system by Wang et. al [28]. Recently, multipartite entanglement in non-inertial frames has been investigated [29-32], where it is shown that entanglement is degraded by the acceleration of the inertial observers. Its extension to the case of a qubit-qutrit system in non-inertial frames under decoherence can be seen in ref. [33]. It is shown that ESB occurs in case of depolarizing channel.

In this letter, the effect of decoherence is investigated for a multipartite system in non-inertial frames by using phase damping and phase flip channels. Three observers Alice, Bob and Charlie share a GHZ type state in non-inertial frames. Alice is considered to be stationary whereas the other two observers move with a uniform acceleration. One-tangles and π-tangles are calculated and discussed for both the channels. Two important features of entanglement, ESD and ESB are investigated.

Let the three observers: Alice, an inertial observer, Bob and Charlie, the accelerated observers moving with uniform acceleration, share the following maximally entangled GHZ state

$$|\Psi\rangle_{ABC} = \frac{1}{\sqrt{2}}\left(|0_{\omega_a}\rangle_A|0_{\omega_b}\rangle_B|0_{\omega_c}\rangle_C + |1_{\omega_a}\rangle_A|1_{\omega_b}\rangle_B|1_{\omega_c}\rangle_C\right) \tag{1}$$

where $|0_{\omega_{a(bc)}}\rangle_{A(BC)}$ and $|1_{\omega_{a(bc)}}\rangle_{A(BC)}$ are vacuum states and the first excited states from the perspective of an inertial observer. Let the Dirac fields, as shown in Refs. [27, 34-35], from the perspective of the uniformly accelerated observers, are described as an entangled state of two modes

monochromatic with frequency ω_i, $\forall_i$

$$|0_{\omega_i}\rangle_M = \cos r_i |0_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II} + \sin r_i |1_{\omega_i}\rangle_I |1_{\omega_i}\rangle_{II} \tag{2}$$

and the only excited state is

$$|1_{\omega_i}\rangle_M = |1_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II} \tag{3}$$

where $\cos r_i = (e^{-2\pi\omega c/a_i} + 1)^{-1/2}$, a_i is the acceleration of i^{th} observer. The subscripts I and II of the kets represent the Rindler modes in region I and II, respectively, in the Rindler spacetime diagram (see Ref. [28], Fig. (1)). Considering that an accelerated observer in Rindler region I has no access to the field modes in the causally disconnected region II and by taking the trace over the inaccessible modes, one obtains the following tripartite state

$$\begin{aligned}|\Psi\rangle_{ABC} \ =\ & \frac{1}{\sqrt{2}}[\cos r_b \cos r_c |0\rangle_A|0\rangle_B|0\rangle_C + \cos r_b \sin r_c |0\rangle_A|0\rangle_B|1\rangle_C \\ & + \sin r_b \cos r_c |0\rangle_A|1\rangle_B|0\rangle_C + \sin r_b \sin r_c |0\rangle_A|1\rangle_B|1\rangle_C + |0\rangle_A|1\rangle_B|1\rangle_C]\end{aligned} \tag{4}$$

For the sake of simplicity, the frequency subscripts are dropped and in density matrix formalism, the above state can be written as

$$\begin{aligned}\rho_{AB_ICI} \ =\ & \frac{1}{\sqrt{2}}[\cos r_b^2 \cos r_c^2 |000\rangle\langle 000| + \cos r_b^2 \sin r_c^2 |001\rangle\langle 001| \\ & + \sin r_b^2 \cos r_c^2 |010\rangle\langle 010| + \sin r_b^2 \sin r_c^2 |011\rangle\langle 011| \\ & + \cos r_b \cos r_c (|000\rangle\langle 111| + |111\rangle\langle 000|) + |111\rangle\langle 111|]\end{aligned} \tag{5}$$

In order to simplify the calculations, it is assumed that Bob and Charlie move with the same acceleration, i.e. $r_b = r_c = r$. The well known entanglement measure for a bipartite system, the negativity can be defined as [36]

$$\mathcal{N}_{AB} = \left\| \rho_{AB}^{T_\alpha} \right\| - 1 \tag{6}$$

where T_α is the partial transpose of ρ_{AB} and $\|.\|$ is the trace norm of a matrix. Whereas for a 3-qubit system $|\Psi\rangle_{ABC}$, $\mathcal{N}_{AB}$ defines the two-tangle which is the negativity of the mixed state $\rho_{AB} = Tr_C(|\Psi\rangle_{ABC}\langle\Psi|)$ and its one-tangle can be defined by

$$\mathcal{N}_{A(BC)} = \left\| \rho_{ABC}^{T_\alpha} \right\| - 1 \tag{7}$$

and the π-tangle is given by

$$\pi_{ABC} = \frac{1}{3}(\pi_A + \pi_B + \pi_C) \tag{8}$$

where $\pi_{A(BC)}$ is the residual entanglement and is defined as

$$\pi_A = \mathcal{N}^2_{A(BC)} - \mathcal{N}^2_{AB} - \mathcal{N}^2_{AC} \tag{9}$$

The interaction between the system and its environment introduces the decoherence to the system, which is a process of the undesired correlation between the system and the environment. The evolution of a state of a quantum system in a noisy environment can be described by the super-operator Φ in the Kraus operator representation as [37]

$$\rho_f = \Phi(\rho_i) = \sum_k E_k \rho_i E_k^\dagger \tag{10}$$

where the Kraus operators E_i satisfy the following completeness relation

$$\sum_k E_k^\dagger E_k = I \tag{11}$$

We have constructed the Kraus operators for the evolution of the tripartite system from the single qubit Kraus operators by taking their tensor product over all n^3 combinations of $\pi(i)$ indices

$$E_k = \underset{\pi}{\otimes} e_{\pi(i)} \tag{12}$$

where n correspond to the number of Kraus operators for a single qubit channel. The single qubit Kraus operators for phase damping channel are given by

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{p} \end{bmatrix} \tag{13}$$

and for phase flip channel

$$E_0 = \sqrt{1-p}I, \quad E_1 = \sqrt{p}\sigma_z \tag{14}$$

where σ_z represents the usual Pauli matrix. Using equations (7-14) along with the initial density matrix of as given in equation (5), the one-tangles and π-tangles of the tripartite system under different environments can be found as given in the following subsections.

The three one-tangles, influenced by the phase damping channel can be calculated by using the definition as given in equation (7), are given by

$$\begin{aligned} \mathcal{N}_{A(BC)} &= -\frac{1}{2} + \frac{1}{2}\cos^4 r + \frac{1}{2}((1-p_0)(1-p_1)(1-p_2)\cos^4 r)^{1/2} \\ &+ \frac{1}{2}((1-p_0)(1-p_1)(1-p_2)\cos^4 r + \sin^8 r)^{1/2} + \frac{1}{4}\sin(2r)^2 \end{aligned} \tag{15}$$

and

$$\begin{aligned}
\mathcal{N}_{B(AC)} &= \mathcal{N}_{C(AB)} = \\
&-\frac{1}{16}+\frac{1}{2}((1-p_0)(1-p_1)(1-p_2)\cos^4 r)^{1/2}+\frac{1}{16}\cos(4r) \\
&+\frac{1}{8}((16-16p_0)(1-p_1)(1-p_2)\cos^4 r+\sin^4(2r))^{1/2} \qquad (16)
\end{aligned}$$

where p_0, p_1 and p_2 are the decoherence parameters corresponding to local coupling of the channel with the qubits of Alice, Bob and Charlie, respectively. The collective coupling corresponds to the situation when $p_0 = p_1 = p_2 = p$. The π-tangle can be calculated easily by using equation (8) and is given by

$$\begin{aligned}
\pi_{ABC} &= \frac{1}{3}(-\frac{1}{2}+\frac{1}{2}\cos^4 r+\frac{1}{2}((1-p_0)(1-p_1)(1-p_2)\cos^4 r)^{1/2} \\
&+\frac{1}{2}((1-p_0)(1-p_1)(1-p_2)\cos^4 r+\sin^8 r)^{1/2}+\frac{1}{4}\sin(2r)^2)^2 \\
&+\frac{2}{3}(-\frac{1}{16}+\frac{1}{2}((1-p_0)(1-p_1)(1-p_2)\cos^4 r)^{1/2}+\frac{1}{16}\cos(4r) \\
&+\frac{1}{8}((16-16p_0)(1-p_1)(1-p_2)\cos^4 r+\sin(2r)^4)^{1/2})^2 \qquad (17)
\end{aligned}$$

The eigenvalues of the partial transpose matrix when only qutrit is influenced by the depolarizing channel are given by

$$\begin{aligned}
\mathcal{N}_{A(BC)} &= -\frac{1}{2}+\frac{1}{2}\cos^2 r(\text{abs}[(1-2p_0)(1-2p_1)(1-2p_2)]+\cos^2 r) \\
&+\frac{1}{2}((1-2p_0)^2(1-2p_1)^2(1-2p_2)^2\cos^4 r+\sin^8 r)^{1/2}+\frac{1}{4}\sin^2(2r)
\end{aligned} \qquad (18)$$

$$\begin{aligned}
\mathcal{N}_{B(AC)} &= \mathcal{N}_{C(AB)} = \\
&-\frac{1}{2}+\frac{1}{2}\text{abs}[(1-2p_0)(1-2p_1)(1-2p_2)]\cos^2 r+1/2\cos^4 r+\frac{1}{2}\sin^4 r \\
&+\frac{1}{8}\sin(2r)^2+\frac{1}{8}(16(1-2p_0)^2(1-2p_1)^2(1-2p_2)^2\cos^4 r+\sin^4(2r))^{1/2}
\end{aligned} \qquad (19)$$

The π-tangle can be calculated easily by using equation (8) and is given by

$$\begin{aligned}
\pi_{ABC} &= \frac{1}{3}(-\frac{1}{2}+\frac{1}{2}\cos^2 r(\mathrm{abs}[(1-2p_0)(1-2p_1)(1-2p_2)]+\cos^2 r) \\
&+\frac{1}{2}((1-2p_0)^2(1-2p_1)^2(1-2p_2)^2\cos^4 r+\sin^8 r)^{1/2}+\frac{1}{4}\sin(2r)^2)^2 \\
&+\frac{2}{3}(-\frac{1}{2}+\frac{1}{2}\mathrm{abs}[(1-2p_0)(1-2p_1)(1-2p_2)]\cos^2 r+\frac{1}{2}\cos^4 r+\frac{1}{2}\sin^4 r \\
&+\frac{1}{8}\sin(2r)^2+\frac{1}{8}(16(1-2p_0)^2(1-2p_1)^2(1-2p_2)^2\cos^4 r+\sin^4(2r))^{1/2})^2
\end{aligned} \tag{20}$$

The two-tangles $\mathcal{N}_{AB}$, $\mathcal{N}_{BC}$ and $\mathcal{N}_{AC}$ can be easily calculated by taking the partial trace of the final density matrix after the environmental effects over qubits C, A and B respectively. All the two-tangles remain zero as expected since the reduced density matrix is not affected by the environment.

Analytical expressions for one-tangles and π-tangles are calculated for a multipartite system in non-inertial frames influenced by phase damping and phase flip environments. The results are consistent with refs. [30, 31] and can be easily verified from the expressions of one-tangles and π-tangles. It is seen that for $r = \pi/4$ all the one-tangles become equal, i.e. $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)}$ for both the environments under consideration. To investigate the effect of decoherence on the multipartite system, the one-tangles and π-tangles are plotted as a function of decoherence parameter, p for different values of acceleration r for phase damping channel in figure 1. Figure 1 (a-c) show the behavior of one-tangles and π-tangles when only Alice's qubit is coupled to the phase damping channel. Whereas figures 1 (d-e) show the behavior of one-tangles and π-tangles when all the three qubits are coupled to the phase damping channel. It is seen that the π-tangles are heavily influenced by the environment as compared to the one-tangles for both the cases (local and collective couplings). Furthermore, as the value of acceleration r, entanglement degradation is enhanced and it is more prominent for higher values of decoherence parameter p. Since, a similar behavior of one-tangles ($\mathcal{N}_{A(BC)}$, and $\mathcal{N}_{B(AC)}$) is seen therefore, only $\mathcal{N}_{A(BC)}$ is plotted for discussion. It is shown that the one-tangles and π-tangles goes to zero at $p = 1$ for phase damping channel. On the other hand, it goes to zero at $p = 0.5$ in case of phase flip channel.

In figure 2, the one-tangles and π-tangles are plotted as a function of decoherence parameter, p for different values of acceleration r influenced by the phase flip channel. Similar to the figure 1, figure 2 (a-c) show the behavior of one-tangles and π-tangles when only Alice's qubit is coupled to the phase flip channel. Whereas figures 2 (d-e) show the behavior of one-tangles and π-tangles when all the three qubits are coupled to the phase flip channel. It is seen that the π-tangles are

heavily influenced by the environment in both the cases (local and collective couplings). It is seen that maximum entanglement degradation occurs at $p = 0.5$ irrespective of the value of acceleration r. It is seen that a sudden entanglement rebound process takes place for $p > 0.5$ in the case of phase flip noise. It is also seen that the one-tangles and π-tangles go to zero at $p = 1$. Since the rebound process is much more stronger than the resistance of acceleration, one cannot ignore it anyway as it is much more prominent for $p > 0.75$. In Fig. 3, the three-dimensional graphs for one-tangles and π-tangles are given as a function of decoherence parameter, p and acceleration r for phase damping and phase flip channels. It is shown that different environments affect the entanglement of the tripartite system differently. It is also seen that in the case of the phase flip channel, ESD behavior becomes independent of the acceleration. However, the sudden death of entanglement cannot be avoided for a phase flip noise around 50% decoherence. Furthermore, entanglement dies out more quickly compared to the phase damping channel for a lower level of decoherence in the case of phase flip noise. The sudden birth and non-vanishing behavior of the one-tangles π-tangle at infinite acceleration is an interesting result. Since Rindler spacetime is similar to Schwarzschild spacetime, it enables one to conjecture that multipartite entanglement does not vanish even if one party falls into the event horizon of the black hole. Hence, some quantum information processing, for example, teleportation, [38] can be performed within and outside the black hole.

In summary, environmental effects on tripartite entanglement in non-inertial frames is investigated by considering a maximally entangled GHZ state shared between three partners. It is assumed that the two partners are in accelerated frames moving with same uniform acceleration. In order to investigate the environmental effects on the entanglement, one-tangles and π-tangles are calculated for the multipartite system. It is seen that in case of phase damping channel, entanglement sudden death (ESD) occurs for higher values of decoherence. Whereas, for the phase flip channel, ESD behaviour happens at $p = 0.5$. In addition, prominent behaviour of entanglement sudden birth (ESB) is seen for $p > 0.75$ in case of phase flip channel. Therefore, it is investigated that the effect of environment is much stronger than that of acceleration for multipartite systems.

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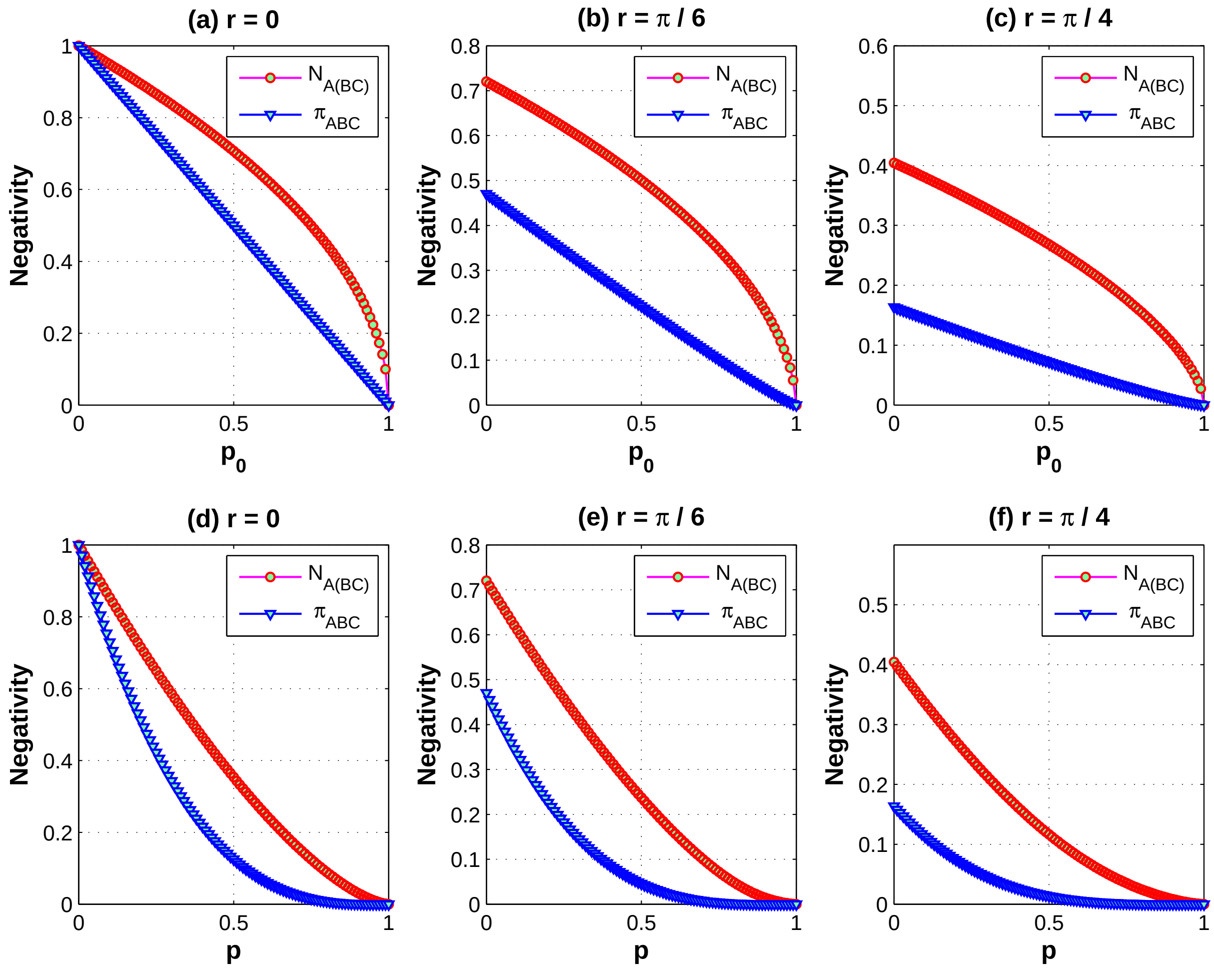

FIG. 1: (Color online). The one-tangles and π-tangles are plotted as a function of decoherence parameter, p for different values of acceleration r for phase damping channel.

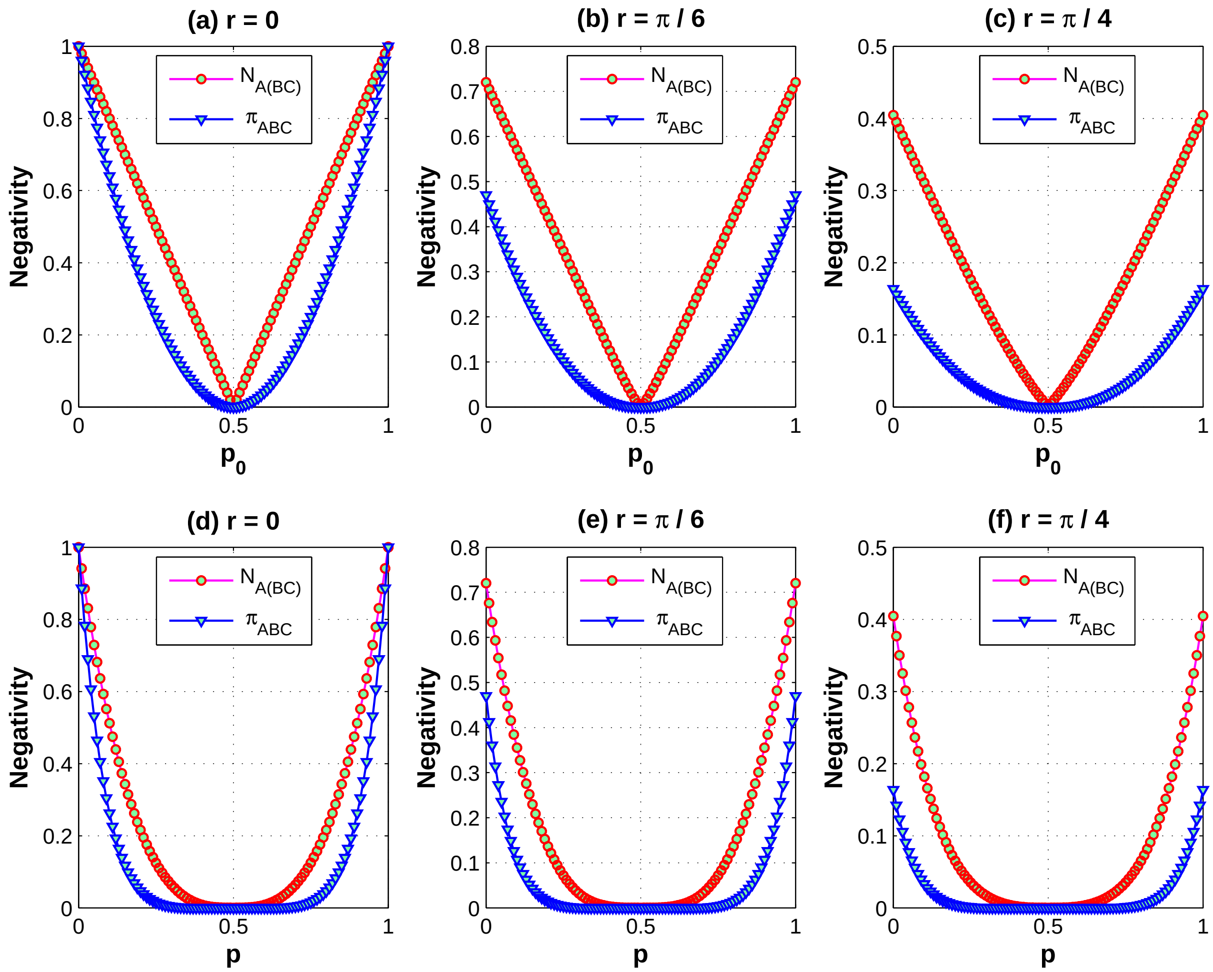

FIG. 2: (Color online). The one-tangles and π-tangles are plotted as a function of decoherence parameter, p for different values of acceleration r for phase flip channel.

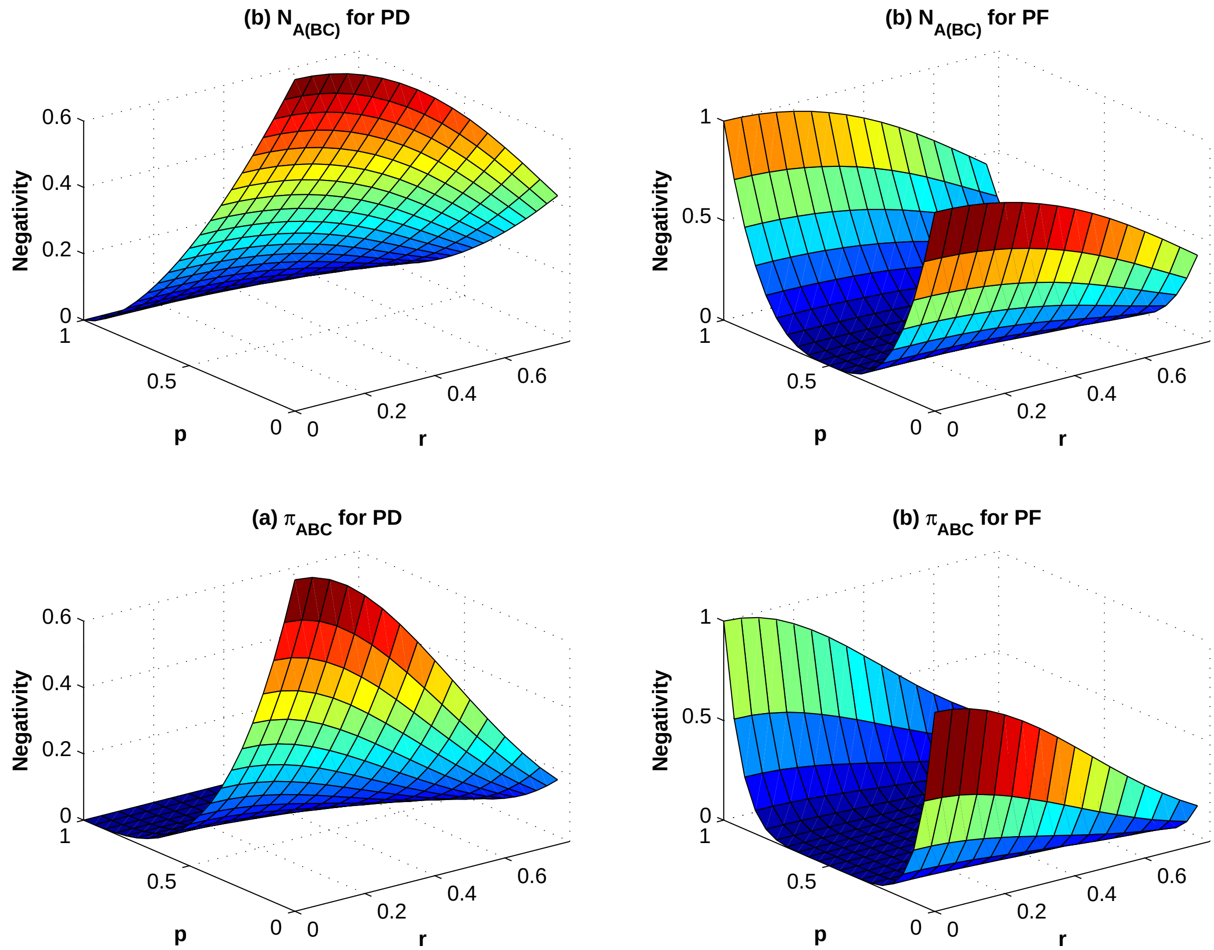

FIG. 3: (Color online). The one-tangles and π-tangles are plotted as a function of acceleration, r and decoherence parameter, p for phase damping and phase flip channels.